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The XY spin glass in the presence of a planar anisotropy field

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Abstract. We consider the infinite-range XY spin glass in the presence of a p-fold anisotropy parameter D_p, favouring p orientations in a plane. This model interpolates between the XY ($D_p \to 0$) and p-state clock ($D_p \to \infty$) spin glasses. For $p \geqslant 4$, D_p is irrelevant in what concerns the qualitative behaviour of the Parisi function. Special attention is devoted to the $p = 3$ case, for which the replica-symmetry-breaking scheme leads to very distinct order-parameter functions in the two end-point limits. We analyse the evolution of the Parisi function for varying D_3 and find two distinct crossovers in its shape. We evaluate the anisotropy parameter (D_3^*) at the crossover and find that the Parisi function changes from its conventional behaviour (continuous and monotonically increasing), typical of the XY spin glass ($D_3 < D_3^*$), to the step function characteristic of Potts spin glasses ($D_3 \geqslant D_3^*$).

1. Introduction

Spin glasses [1, 2] represent one of the most puzzling and controversial subjects in the physics of disordered systems. Although a lot of effort has been concentrated on this problem during the last two decades, many fundamental questions remain unanswered. Most of the progress has been achieved for infinite-range models, the prototype of which is the Ising version introduced by Sherrington and Kirkpatrick (SK) [3]. The simple solution proposed by SK, known as replica-symmetry approximation, was shown to be unstable at low temperatures throughout a considerable part of the phase diagram, including the whole spin-glass phase [4]. The solution now accepted as being correct for this problem, based on a replica-symmetry-breaking scheme, was proposed by Parisi [5] and consists of an infinite number of order parameters, i.e. an order-parameter function. Another striking result of the SK model is the survival of the spin-glass phase transition in the presence of a uniform magnetic field, signalled by the Almeida–Thouless (AT) line [4]. Below the AT line, the existence of many free-energy minima requires the use of Parisi's solution.

One fundamental question, which is far from being answered today, concerns the applicability of the formalism developed for the SK model in the description of three-dimensional spin glasses. An alternative theory has been proposed by Fisher and Huse [6] for short-range spin glasses, the so-called 'droplet model', the main conclusions of which differ radically from those of the SK model. The droplet model, which is based on previous scaling arguments [7], predicts, for any *finite* dimension, a spin-glass state described in terms of a single order parameter, and no phase transition in the presence of a uniform magnetic field. The assumptions of the droplet model have been seriously criticized [8] and recent numerical investigations suggest its failure at $d = 4$ [9–11] but are not conclusive at $d = 3$ [10–12]. However, many experimental investigations associate the onset of strong irreversibility effects with the AT line; this suggests that the SK model may not be appropriate,

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but should be seriously considered in the construction of a better theory for the description of real spin glasses.

Generalizations of the SK model in order to include continuous spin variables (e.g., m-vector spin glasses [13]), or more complex discrete variables (e.g., Potts glasses [14, 15], clock glasses [16–19]), provoked many questions which had not previously arisen. In particular, for m-vectors ($m \geqslant 2$), the transition in a magnetic field is set at a line (invariant under field inversion) signalling the ordering of the transverse degrees of freedom [13], below which the Parisi solution is required [20]; however, the order-parameter functions are qualitatively similar to those of the SK model [21]. For Potts glasses, the line corresponding to the transition in a field may change under field inversion [17], contrary to the m-vector case; the Parisi solution is now quite different from the solution of the SK model [15]. The clock spin glasses present $p = 3$ as a singular case for which the Parisi function [16] and the line corresponding to the transition in a field (which changes under field inversion) is qualitatively distinct from all other cases [17], whereas at $p = 4$ one finds a spontaneously anisotropic spin-glass order [18].

In this paper, we consider the infinite-range XY spin glass in the presence of a p-fold anisotropy parameter D_p. Such a parameter favours p equally spaced orientations in a plane and may simulate the crystal fields in real substances [22], allowing an interpolation between the XY ($D_p \to 0$) and clock ($D_p \to \infty$) spin glasses. In section 2, we introduce the model and find its free-energy density within the replica method; the instability of the replica-symmetric solution is verified for temperatures just below the spin-glass freezing. In section 3, we implement Parisi's parametrization and discuss the possible order-parameter functions; we focus our analysis on the $p = 3$ case, for which the two end points ($D_3 = 0$ and $D_3 \to \infty$) present distinct replica-symmetry-breaking behaviour. Two crossovers are found in the shape of the Parisi function; in particular, we estimate the anisotropy parameter D_3^* at the crossover where the order parameter becomes a step function. Finally, in section 4, we summarize our main results.

2. The model and its free-energy density

Let us consider an XY spin glass defined in terms of the Hamiltonian

$$H = -\sum_{(ij)} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j - D_p \sum_i \cos p\theta_i \tag{2.1}$$

where $\boldsymbol{S}_i$ ($i = 1, 2, \ldots, N$) are classical two-component spin variables

$$\boldsymbol{S}_i \equiv (S_{ix}, S_{iy}) = (\cos\theta_i, \sin\theta_i) \tag{2.2}$$

and D_p is an anisotropy parameter, which may simulate crystal fields in real substances [22], favouring p discrete orientations in the XY plane. The sum $\sum_{(ij)}$ is extended to all distinct pairs of spins, i.e. the interactions are infinite-range-like, with the $\{J_{ij}\}$ being quenched random couplings associated with the probability distribution

$$P(J_{ij}) = \left(\frac{N}{2\pi J^2}\right)^{1/2} \exp\left(-\frac{N}{2J^2} J_{ij}^2\right). \tag{2.3}$$

Such a model interpolates between two well known systems, namely the XY ($D_p \to 0$) [13, 20, 21] and p-state clock ($D_p \to \infty$) [16–19] spin glasses. Throughout this paper we shall restrict ourselves to the cases where $p > 2$.

As usual, the average over the disorder ($[\ ]_J$) is performed by introducing n copies (or replicas) of the original model, in terms of which the free-energy per spin may be written as

$$-\beta f = \lim_{N\to\infty} \frac{1}{N}[\ln Z\{J_{ij}\}]_J = \lim_{N\to\infty}\lim_{n\to 0} \frac{1}{Nn}([Z^n\{J_{ij}\}]_J - 1) \tag{2.4}$$

where $\beta = (k_B T)^{-1}$, T is the temperature and $Z\{J_{ij}\}$ is the canonical partition function associated with a given bond realization $\{J_{ij}\}$. Within the standard procedure of the replica method [1], we shall assume that the limits appearing in (2.4) may be freely interchanged such that they give

$$\beta f = \lim_{n\to 0} \frac{1}{n} \min g(R^\alpha, Q^{\alpha\beta}_{\mu\nu}) \tag{2.5}$$

where

$$g(R^\alpha, Q^{\alpha\beta}_{\mu\nu}) = -\frac{n}{8}(\beta J)^2 + \frac{(\beta J)^2}{2}\sum_\alpha (R^\alpha)^2 + \frac{(\beta J)^2}{2}\sum_{(\alpha\beta)}\sum_{\mu\nu}(Q^{\alpha\beta}_{\mu\nu})^2 - \ln \mathrm{Tr}_\alpha \exp\{H_{\mathrm{eff}}\} \tag{2.6a}$$

$$H_{\mathrm{eff}} = (\beta J)^2 \sum_\alpha R^\alpha[(S^\alpha_x)^2 - \tfrac{1}{2}] + (\beta J)^2 \sum_{(\alpha\beta)}\sum_{\mu\nu} Q^{\alpha\beta}_{\mu\nu} S^\alpha_\mu S^\beta_\nu . \tag{2.6b}$$

In these equations, α and β are replica labels ($\alpha, \beta = 1, 2, \ldots, n$), Tr_α is a trace over the spin variables for each replica α, $\sum_{(\alpha\beta)}$ denote sums over distinct pairs of replicas and $\mu, \nu = x, y$ are Cartesian components.

The extrema of functional (2.6) yield the equations of state

$$R^\alpha = \langle (S^\alpha_x)^2 \rangle - \tfrac{1}{2} \tag{2.7a}$$

$$Q^{\alpha\beta}_{\mu\nu} = \langle S^\alpha_\mu S^\beta_\nu \rangle \qquad (\mu, \nu = x, y; \alpha \neq \beta) \tag{2.7b}$$

where the brackets $\langle\ \rangle$ denote thermal averages with respect to the 'effective Hamiltonian' H_{eff}.

One may estimate the critical temperatures $T_c^{(R)}$ and $T_c^{(Q)}$, associated with the parameters R^α and $Q^{\alpha\beta}_{\mu\nu}$, respectively, by expanding the free-energy functional in a power series and analysing their corresponding quadratic terms. By doing so, one finds (in units $k_B = 1$)

$$T_c^{(R)} = \frac{J}{2}\left[\frac{1}{2}\left(1 + \frac{I_1}{I_0}\delta_{4,p}\right)\right]^{1/2} \tag{2.8a}$$

$$T_g = T_c^{(Q)} = \tfrac{1}{2}J \qquad (\forall D_p) \tag{2.8b}$$

where $I_k = I_k(\beta D_p)$ ($k = 0, 1$) denote modified Bessel functions of the first kind, of order k. One sees that, except for the pathological case $p = 4$, $D_4 \to \infty$ (four-state clock) discussed elsewhere [18, 19], the spin-glass critical temperature $T_g (\equiv T_c^{(Q)})$ is always greater than $T_c^{(R)}$. Therefore, the correct temperature associated with a *possible* spontaneous quadrupolar ordering is not simply given by (2.8a) and must be re-evaluated, taking into account the presence of the spin-glass parameters.

Throughout the rest of this paper we shall restrict ourselves to a small temperature range, just below T_g, i.e. small

$$\tau = 1 - \frac{T}{T_g} = 1 - \frac{2T}{J}. \tag{2.9}$$

Within this temperature range one may assume the isotropic conditions [1]

$$R^\alpha = 0 \tag{2.10a}$$

$$Q^{\alpha\beta}_{\mu\nu} = Q^{\alpha\beta}\delta_{\mu\nu} \tag{2.10b}$$

so that we may express the functional in (2.6) as a power series depending on $Q^{\alpha\beta}$ only:

$$\begin{aligned} g(Q^{\alpha\beta}) = -ng_0 - a_2\sum_{\alpha\beta}(Q^{\alpha\beta})^2 - a_3\sum_{\alpha\beta}(Q^{\alpha\beta})^3 - b_3\sum_{\alpha\beta\gamma} Q^{\alpha\beta}Q^{\beta\gamma}Q^{\gamma\alpha} - a_4\sum_{\alpha\beta}(Q^{\alpha\beta})^4 \\ - b_4\sum_{\alpha\beta\gamma}(Q^{\alpha\beta})^2(Q^{\beta\gamma})^2 - c_4\sum_{\alpha\beta\gamma} Q^{\alpha\beta}Q^{\beta\gamma}(Q^{\gamma\alpha})^2 \\ - d_4\sum_{\alpha\beta\gamma\delta} Q^{\alpha\beta}Q^{\beta\gamma}Q^{\gamma\delta}Q^{\delta\alpha} - \cdots \end{aligned} \tag{2.11}$$

with

$$\begin{aligned} g_0 &= \frac{(\beta J)^2}{8} + \ln(2\pi I_0) \qquad a_2 = \frac{(\beta J)^2}{2}\left[\frac{(\beta J)^2}{4} - 1\right] \\ a_3 &= \frac{(\beta J)^6}{48}\left(\frac{I_1}{I_0}\right)^2 \delta_{3,p} \qquad b_3 = \frac{(\beta J)^6}{24} \qquad a_4 = \frac{(\beta J)^8}{384}\left[3 + \left(\frac{I_1}{I_0}\right)^2 \delta_{4,p}\right] \\ b_4 &= -\frac{(\beta J)^8}{32} \qquad c_4 = \frac{(\beta J)^8}{32}\left(\frac{I_1}{I_0}\right)^2 \delta_{3,p} \qquad d_4 = \frac{(\beta J)^8}{64}. \end{aligned} \tag{2.12}$$

We are now confronted with the central question of the replica method for spin glasses, which is how to choose a parametrization for the matrix elements $Q^{\alpha\beta}$ such that we obtain reasonable (i.e. thermodynamically acceptable) results in the $n \to 0$ limit. The simplest choice is known as replica-symmetry approximation [3] and consists in taking

$$Q^{\alpha\beta} = Q \qquad (\forall \text{ pairs } (\alpha\beta)). \tag{2.13}$$

However, such a choice leads to problems in the spin-glass phase, i.e. the stability matrix [4]

$$S^{(\alpha\beta)(\gamma\delta)} = \frac{\partial^2 g(Q^{\alpha\beta})}{\partial Q^{\alpha\beta}\partial Q^{\gamma\delta}} \tag{2.14}$$

presents an eigenvalue

$$\lambda = P - 2V + R \tag{2.15a}$$

$$P = S^{(\alpha\beta)(\alpha\beta)} \tag{2.15b}$$

$$V = S^{(\alpha\beta)(\alpha\gamma)} \qquad (\beta \neq \gamma) \tag{2.15c}$$

$$R = S^{(\alpha\beta)(\gamma\delta)} \qquad (\alpha, \beta \neq \gamma, \delta) \tag{2.15d}$$

which, when evaluated within (2.13), becomes negative in the spin-glass phase, signalling the instability of the replica-symmetric solution. Indeed, for τ small, one may show that

$$\begin{aligned}\lambda &= -6a_3 Q - 16a_4 Q^2 + O(Q^3)\\ &= -\left[\frac{(\beta J)^6}{8}\left(\frac{I_1}{I_0}\right)^2 \delta_{3,p}\right] Q - \frac{(\beta J)^8}{24}\left[3 + \left(\frac{I_1}{I_0}\right)^2 \delta_{4,p}\right] Q^2 + O(Q^3)\end{aligned} \tag{2.16}$$

where $Q \sim O(\tau)$. Therefore, for any $p > 3$, the AT instability occurs at $O(\tau^2)$, as in the m-vector spin glasses [13, 20]. However, if $p = 3$, this instability appears with two different τ-dependences: $\lambda \sim O(\tau^2)$ (when $D_3 \to 0$) and $\lambda \sim O(\tau)$ (when $D_3 \to \infty$), the crossover between them occurring at $(D_3/J)^2 \sim \tau$. Therefore, if $(D_3/J)^2 \gg \tau$, the AT instability shows up in a stronger form, as in the Potts glass [14].

In the next section we will discuss the parametrization which is now accepted to be correct for infinite-range spin glasses.

3. Replica-symmetry breaking

Parisi's proposal for replica-symmetry breaking [5] consists of a hierarchical process in which the order-parameter matrix **Q** (elements $Q^{\alpha\beta}$) is split into blocks, with all elements in a given block the same; the process is repeated successively for the diagonal blocks and at each step new order parameters are introduced. Parisi showed that in the $n \to 0$ limit, the matrix **Q** is associated with an order-parameter function $Q(x)$ defined in the interval [0,1]. The procedure may easily be carried out for small τ, such that from (2.11) one obtains the free-energy functional

$$\begin{aligned}\beta f[Q] = &-g_0 + a_2\langle Q^2\rangle + a_3\langle Q^3\rangle - b_3 \int_0^1 \mathrm{d}x \left[x Q^3(x) + 3Q(x)\int_0^x Q^2(y)\,\mathrm{d}y\right]\\ &+ a_4\langle Q^4\rangle + b_4\left\{\langle Q^4\rangle - 2\langle Q^2\rangle^2 - \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y\,[Q^2(x) - Q^2(y)]^2\right\}\\ &- c_4\left\{2\langle Q\rangle\langle Q^3\rangle + \int_0^1 \mathrm{d}x\, Q^2(x)\int_0^x \mathrm{d}y\,[Q(x) - Q(y)]^2\right\}\\ &- d_4\left\{\langle Q^2\rangle^2 - 4\langle Q^2\rangle\langle Q\rangle^2 - \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \int_0^x \mathrm{d}z\,[Q(x)-Q(y)]^2[Q(x)-Q(z)]^2\right.\\ &\left. - 4\langle Q\rangle \int_0^1 \mathrm{d}x\, Q(x)\int_0^x \mathrm{d}y\,[Q(x) - Q(y)]^2\right\} - \cdots\end{aligned} \tag{3.1}$$

where $\langle Q^m\rangle = \int_0^1 \mathrm{d}x\, Q^m(x)$. One follows the usual recipe for determining the function $Q(x)$ by applying successive derivatives to the extremum condition

$$\frac{\delta(\beta f[Q])}{\delta Q} = 0 \tag{3.2}$$

to determine the slope ($Q'(x)$), the height of the plateau (Q_m) and the breaking point (x_1). One obtains

$$Q'(x) = \frac{1}{3}\left[1 - \frac{3}{2}\left(\frac{I_1}{I_0}\right)^4 \delta_{3,p} + \frac{1}{3}\left(\frac{I_1}{I_0}\right)^2 \delta_{4,p}\right]^{-1} + O(\tau) \tag{3.3a}$$

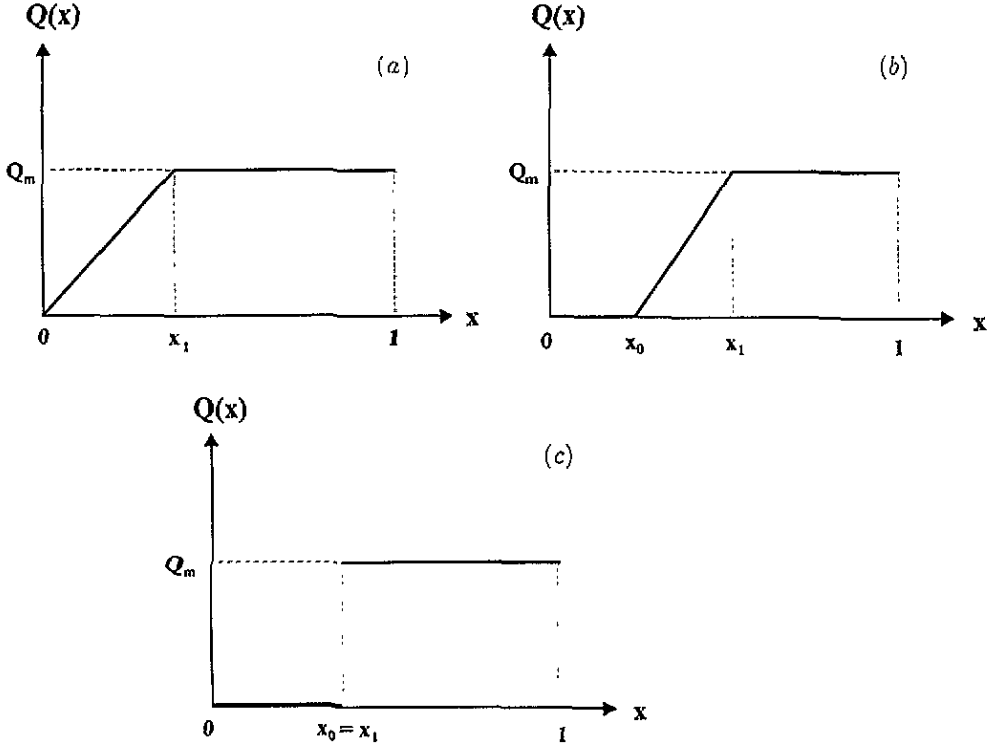

Figure 1. The order-parameter function $Q(x)$ to lowest order in τ: (a) the XY regime; (b) $p = 3$ and $D_3 < D_3^*$ (such that $(D_3/J)^2 \gg \tau$)), crossover 1 (from shapes (a) to (b)) occurs for $(D_3/J)^2 \sim \tau$); (c) $p = 3$ and $D_3 \geqslant D_3^*$, crossover 2 (from shapes (b) to (c)) occurs at an anisotropy parameter for which the slope $Q'(x)$ diverges, $D_3^* \cong 2.75J$.

for $x_0 < x < x_1$ and

$$Q_m = \frac{1}{2}\left[1 - \frac{1}{4}\left(\frac{I_1}{I_0}\right)^2 \delta_{3,p}\right]^{-1} \tau + O(\tau^2) \tag{3.3b}$$

for $x_1 \leqslant x \leqslant 1$, where

$$x_0 = \frac{1}{2}\left(\frac{I_1}{I_0}\right)^2 \delta_{3,p} + O(\tau^2) \tag{3.4a}$$

$$x_1 = x_0 + \frac{Q_m}{Q'(x)} + O(\tau^2) \tag{3.4b}$$

with $Q(x) = 0$ for $x \leqslant x_0$.

To the lowest order in τ, all cases $p \geqslant 5$ are independent of D_p, such that $p = 5$ is already in the XY regime with (see figure 1(a))

$$x_0 = O(\tau^2) \qquad x_1 = \tfrac{3}{2}\tau + O(\tau^2)$$

$$Q'(x) = \tfrac{1}{3} + O(\tau) \qquad Q_m = \tfrac{1}{2}\tau + O(\tau^2) \qquad (\forall D_p). \tag{3.5}$$

The order-parameter function is qualitatively similar to the order-parameter function of the SK model. The case $p = 4$ presents a similar function $Q(x)$, with a slope which varies with D_4, although the qualitative shape displayed in figure 1(a) remains the same.

An interesting behaviour may be found in the case $p = 3$, for which

$$Q'(x) = \frac{1}{3}\left[1 - \frac{3}{2}\left(\frac{I_1}{I_0}\right)^4\right]^{-1} + O(\tau) \tag{3.6a}$$

$$Q_m = \frac{1}{2}\left[1 - \frac{1}{4}\left(\frac{I_1}{I_0}\right)^2\right]^{-1} \tau + O(\tau^2) \tag{3.6b}$$

$$x_0 = \frac{1}{2}\left(\frac{I_1}{I_0}\right)^2 + O(\tau^2) \tag{3.6c}$$

$$x_1 = x_0 + \frac{Q_m}{Q'(x)} + O(\tau^2). \tag{3.6d}$$

One sees that the slope $Q'(x)$ goes from a positive to a negative value (diverging in between), such that $Q(x)$ changes radically in shape as D_3 varies in the interval $[0, \infty]$.

For D_3 small, one obtains

$$Q'(x) = \frac{1}{3} + \frac{1}{2}\left(\frac{D_3}{J}\right)^4 + O(\tau) \tag{3.7a}$$

$$Q_m = \frac{1}{2}\tau + \frac{1}{8}\left(\frac{D_3}{J}\right)^2 \tau + O(\tau^2) \tag{3.7b}$$

$$x_0 = \frac{1}{2}\left(\frac{D_3}{J}\right)^2 + O(\tau^2) \tag{3.7c}$$

$$x_1 = x_0 + \frac{3}{2}\left[1 + \frac{1}{4}\left(\frac{D_3}{J}\right)^2 - \frac{3}{2}\left(\frac{D_3}{J}\right)^4\right]\tau + O(\tau^2). \tag{3.7d}$$

In this limit, if $(D_3/J)^2 \ll \tau$, one obtains, as a first approximation, the XY-like function specified in equation (3.5) and shown in figure 1(a); however, for $(D_3/J)^2 \gg \tau$, one obtains a non-neglectable value for x_0 and the shape presented in figure 1(b). The crossover between figures 1(a) and 1(b) occurs for $(D_3/J)^2 \sim \tau$; this may be correlated to the crossover found through the τ-dependence of the 'dangerous eigenvalue' in the AT instability discussed in section 2. From now on we shall refer to this as crossover 1.

As the anisotropy D_3 increases further, the slope $Q'(x)$ grows faster than the plateau height Q_m, such that $(x_1 - x_0)$ decreases. One may even obtain a regime with $Q'(x)$ negative, which is forbidden within Parisi's theory [1, 2]. For this regime, a different order-parameter function will be considered, i.e. the step-function previously proposed for the Potts glass [15], as shown in figure 1(c). One may define a crossover (crossover 2) between the shapes displayed in figure 1(b) and 1(c); this occurs at an anisotropy parameter $D_3 = D_3^*$, such that as $Q'(x) \to \infty$

$$D_3^* \cong 2.75J + O(\tau) \tag{3.8}$$

corresponding to $x_0^* \cong 0.4082 + O(\tau^2)$, $x_1^* = x_0^* + O(\tau^2)$ and $Q_m^* \cong 0.6282\tau + O(\tau^2)$. For $D_3 > D_3^*$, there is no qualitative change in $Q(x)$, although x_1 and Q_m vary continuously in the intervals $[x_1^*, \frac{1}{2}]$ and $[Q_m^*, \frac{2}{3}\tau]$, respectively, for $D_3^* \leqslant D_3 < \infty$. The order-parameter function for the three-state Potts spin glass [15, 16] is recovered when $D_3 \to \infty$.

4. Conclusion

We have studied an infinite-range XY spin glass in the presence of a p-fold anisotropy field D_p, favouring p ($p > 2$) equally spaced directions in a plane. Within this model, one may interpolate between the XY ($D_p \to 0$) and p-state clock ($D_p \to \infty$) spin glasses.

We have concentrated our analysis on a temperature range near the spin-glass freezing $T_g = J/2$ (J is the width of the couplings Gaussian distribution), i.e. τ small ($\tau = 1 - T/T_g$). The AT instability of the replica-symmetric solution was verified for all values of p and D_p. We have shown that the 'dangerous eigenvalue' of the stability matrix becomes negative at $O(\tau^2)$ for all $p \geqslant 4$, as in the SK model. However, for $p = 3$, a crossover (crossover 1) occurs in the τ-dependence of such an eigenvalue for $(D_3/J)^2 \sim \tau$, where it goes from the usual $O(\tau^2)$ ($(D_3/J)^2 \ll \tau$) to $O(\tau)$ ($(D_3/J)^2 \gg \tau$), signalling a stronger instability.

We implemented Parisi's replica-symmetry breaking ansatz to this model. For any $p \geqslant 4$, the same SK-like qualitative behaviour of the order-parameter function $Q(x)$ was obtained for $D_p \in [0, \infty]$. However, in the case $p = 3$, an interesting evolution in the Parisi function was observed for increasing D_3. A first crossover (crossover 1), which may be correlated to the cossover found by AT stability analysis, was verified for $(D_3/J)^2 \sim \tau$, where $Q(x) = 0$ departs from $x = 0$. A second crossover (crossover 2) was obtained at a value $D_3^* \cong 2.75J$ for which $Q(x)$ changes from a monotonically increasing function to a step function characteristic of Potts glasses.

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